

**TURKISH  
AVIATION  
ACADEMY**



**İTÜ**



## ***Fleet Assignment***

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### ***Dr. Peter Belobaba***

***Istanbul Technical University***  
***Air Transportation Management***  
***M.Sc. Program***

***Network, Fleet and Schedule***  
***Strategic Planning***  
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## *Lecture Outline*

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- **Fleet Assignment Problem**
  - Objectives and principal economic trade-offs
- **Single Flight Leg vs. Network Fleet Assignment**
  - Operational constraints and modeling assumptions
  - Example: Network fleet assignment problem
- **Network Fleet Assignment Optimization**
  - Objective function and constraints
  - Solution times
- **Coldstart: Fleet Assignment at Delta Air Lines**

# ***SCHEDULE DEVELOPMENT***

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- **Involves several interrelated decisions, which to date have not been fully integrated:**

Frequency Planning: Number of departures to be offered on each route, non-stop versus multi-stop

Timetable Development: Flight departure and arrival times, including connections at airline hubs

Fleet Assignment: Aircraft type for each flight, based on demand and operating cost estimates

Aircraft Rotation Planning: Links consecutive flights to ensure balanced aircraft flows on the network.

## ***Fleet Assignment Problem***

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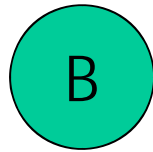
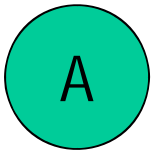
- **Given a schedule of flight legs (origin, destination, departure and arrival times), as well as:**
  - Number of Aircraft by Equipment Type
  - Turn Times by Fleet Type at each Station
  - Other Restrictions: Maintenance, Gate, Noise, Runway, etc.
- **Operating Costs and Spill Costs determine the Total Potential Contribution of each Flight, by Fleet Type**
- **What is the optimal (contribution/profit maximizing) assignment of aircraft to each flight leg?**

## ***Trade-offs in Fleet Assignment***

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- **Operating costs increase with size of airplane for any given flight (typically)**
  - Larger aircraft have higher ownership and maintenance costs
  - Increased fuel burn with greater capacity and weight
  - More (and perhaps higher paid) crew members required
- **Spill costs decrease with size of airplane**
  - SPILL is rejected demand due to inadequate capacity
  - Larger aircraft accommodate more demand and generate more revenue, meaning less spill and lower spill costs
- **Economic trade-off in choosing optimal fleet type**
  - Too large an aircraft leads to higher costs, empty seats
  - Too small an aircraft leads to higher load factors but more rejected demand and lost revenue potential

## Fleet Assignment Example – Single Leg



Demand = 100

Fare = \$100

Fleet Type	Capacity	Spill Cost	Op. Cost	Assignment Cost
i	80	\$2,000	\$5,000	\$7,000
ii	100	\$0	\$6,000	\$6,000
iii	120	\$0	\$7,000	\$7,000
iv	150	\$0	\$8,000	\$8,000

## ***Single Leg vs. Network Fleet Assignment***

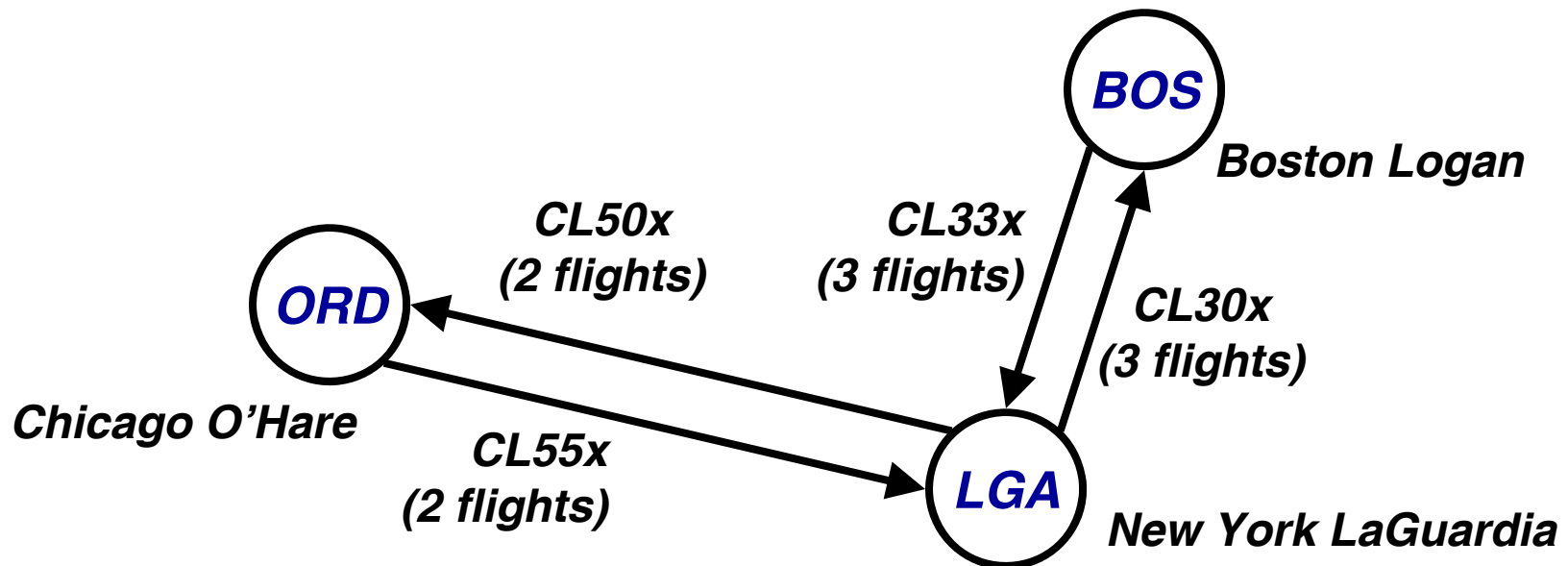
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- **Single leg spill and fleet assignment is unrealistic:**
  - Larger aircraft must be available at origin airport at required departure time
  - Larger aircraft must return (or continue onward) from destination airport
  - Smaller aircraft must be assigned to an alternative profitable flight leg
  - Crew rotations, maintenance and other considerations
- **Also, single leg model assumes 100% local traffic**
  - Changing aircraft size on one flight will affect connecting network passenger flows on other flights
  - No recapture assumed – changing aircraft size on one flight will affect passenger loads on other flights on the same route

## Example: Network Fleet Assignment

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### Flight Network





## *Example: Network Fleet Assignment*

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### **Flight Schedule, Fares, & Demand**

<b>Flight #</b>	<b>From</b>	<b>To</b>	<b>Dept Time (EST)</b>	<b>Arr Time (EST)</b>	<b>Fare [\$]</b>	<b>Demand [passengers]</b>
CL301	LGA	BOS	1000	1100	150	250
CL302	LGA	BOS	1100	1200	150	250
CL303	LGA	BOS	1800	1900	150	100
CL331	BOS	LGA	0700	0800	150	150
CL332	BOS	LGA	1030	1130	150	300
CL333	BOS	LGA	1800	1900	150	150
CL501	LGA	ORD	1100	1400	400	150
CL502	LGA	ORD	1500	1800	400	200
CL551	ORD	LGA	0700	1000	400	200
CL552	ORD	LGA	0830	1130	400	150

## *Example: Network Fleet Assignment*

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### **Fleet Information**

<b>Fleet type</b>	<b>Number of aircraft owned</b>	<b>Capacity [seats]</b>	<b>Per flight operating cost [\$000]</b>	
			<b>LGA - BOS</b>	<b>LGA – ORD</b>
DC-9	1	120	10	15
B737	2	150	12	17
A300	2	250	15	20

## ***Example: Network Fleet Assignment***

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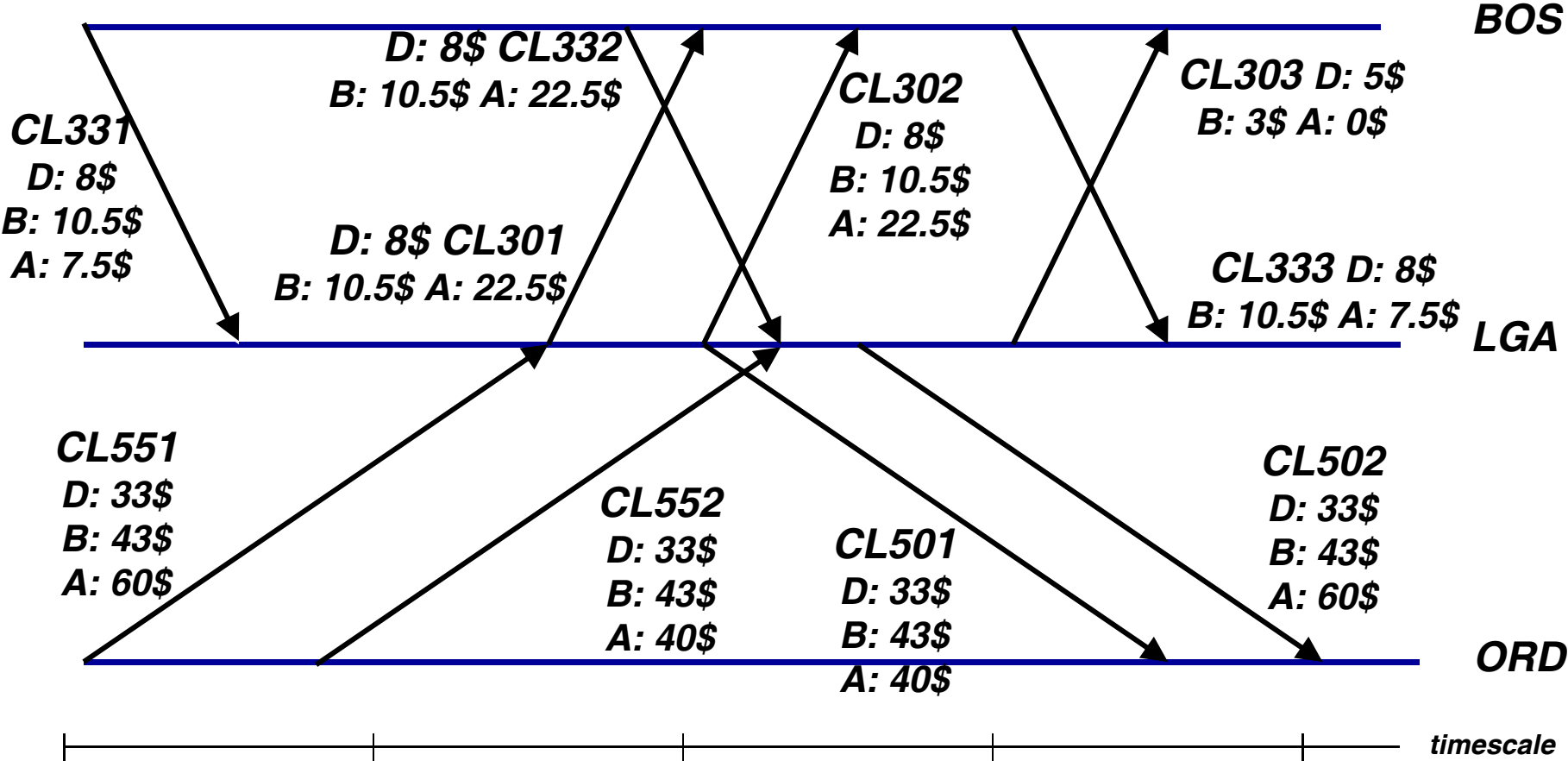
### ***Evaluating assignment profits...***

***Profitability [\$000 per day]***

<b>Flight #</b>	<b>DC-9</b>	<b>B737</b>	<b>A300</b>
CL301	8	10.5	22.5
CL302	8	10.5	22.5
CL303	5	3	0
CL331	8	10.5	7.5
CL332	8	10.5	22.5
CL333	8	10.5	7.5
CL501	33	43	40
CL502	33	43	60
CL551	33	43	60
CL552	33	43	40

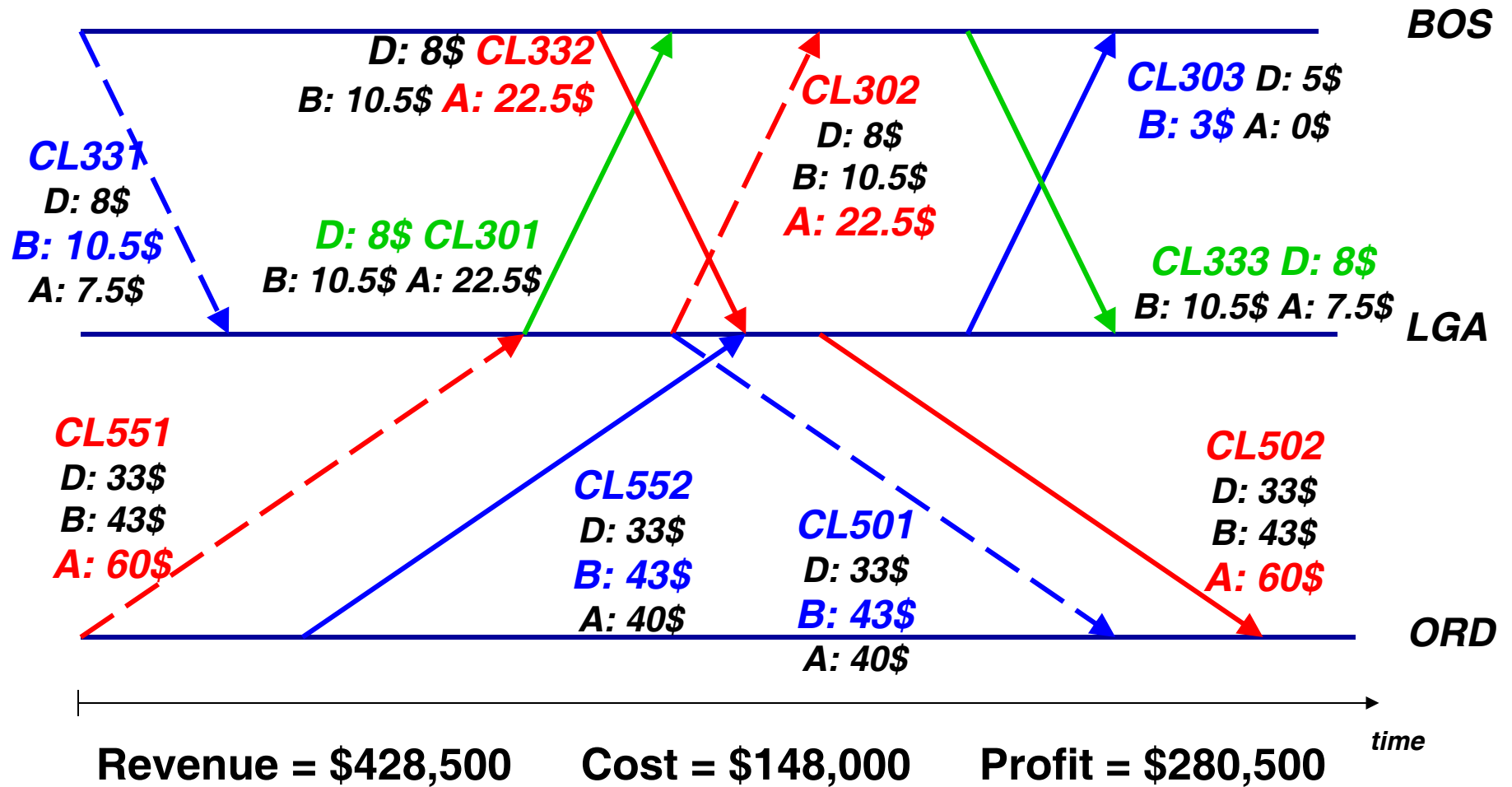
# Example: Network Fleet Assignment

Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:



# Fleet Assignment Solution

Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:



## ***Network Fleet Assignment Objective Function***

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- **For each fleet type/flight combination:**

**Assignment Cost  $\equiv$  Operating cost + Spill cost**

- **Operating cost of assigning a fleet type  $k$  to a flight leg  $j$  is relatively straightforward to compute**
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning “infinite” costs
- **Spill cost for flight leg  $j$  and fleet assignment  $k$** 
  - Average revenue per passenger on  $j$  \* MAX(0, unconstrained demand for  $j$  – number of seats on  $k$ )
  - But revenue for each flight leg is affected by fare class mix (RM) as well as itinerary mix in a network

## *Constraints*

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- **Cover Constraints**
  - Each flight must be assigned to exactly one fleet type
- **Balance Constraints**
  - Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing
- **Aircraft Count Constraints**
  - Number of aircraft of a fleet type used cannot exceed the number available

# *Network Fleet Assignment Formulation*

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## Formulation: The Fleet Assignment Model (FAM)

maximize

***PROFIT***

subject to

***Flight Cover***

***Aircraft Balance***

***Aircraft Count***

***Integrality and Non-negativity***

*Ref.: Hane C., C. Barnhart, E. Johnson, R. Marsten, G. Nemhauser, G. Sigismondi. 1995. The Fleet Assignment Problem: Solving a Large-Scale Integer Program. Mathematical Programming 70 211-232.*



## ***Solution***

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- **Solve fleet assignment problems for large network carriers (10-14 fleets, 2000-3500 flights) within 10-20 minutes of computation time on workstation class computers**
- **Hane, et al. “The Fleet Assignment Problem, Solving a Large Integer Program,” *Mathematical Programming*, Vol. 70, 2, pp. 211-232, 1995**

## ***COLDSTART: Fleet Assignment at Delta Air Lines***

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- **1994 Interfaces article describes implementation of large-scale network fleet assignment optimization**
  - 2500 domestic flight legs per day
  - 450 aircraft of 10 different fleet types
- **Mixed-integer linear program**
  - Minimize assignment costs over the Delta domestic network and schedule for one day
  - Assigns fleet types to each leg, not tail numbers (aircraft routing performed subsequently)
- **First OR application of this size implemented at Delta**
  - Use of this model estimated to increase operating profit by \$100 million per year

## ***Constraints and Issues in Coldstart***

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- **Certain pairs of legs must be assigned same fleet**
  - Provide one-stop, same-plane service through the hub
  - Tag-end flights that must be operated with same aircraft (e.g., IST-GRU-EZE)
- **Model includes maintenance requirements**
  - Use “maintenance arcs” to represent flights that must be covered with an aircraft
  - For example, a B757 must be at a designated base each night
- **Crew considerations**
  - Common fleet families use same pilot aggregates
  - Penalize fleet assignments that require extended crew rest periods – for example, when only 1 flight into/out of a city is assigned an aircraft type

## ***Constraints and Issues in Coldstart***

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- **Aircraft performance can differ within each fleet type**
  - Different engines; take-off and landing weights
  - Not all aircraft equipped to serve over-water routes
- **Airport characteristics and restrictions**
  - Runway lengths and temperature limitations
  - Certain airports have noise restrictions and/or curfews for specific aircraft types
- **Assumed turn times determine aircraft availability**
  - Minimum turn around times vary by both aircraft type and airport
  - Larger aircraft require longer turn times
  - International flights require longer turn times than domestic flights with same fleet type

## ***Coldstart in the Scheduling Process***

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- **Changes to the functions of DL schedule planners**
  - Model performs assignments to schedule, while planner reviews and analyzes the impacts of assignment changes
  - Focus on cost and revenue inputs to the model instead of the actual optimization process
- **Model allows for what-if analysis by planners**
  - Compare results from two different optimization runs to compare impacts of various costs and constraints
  - Evaluate the changes required in order to change the aircraft assigned to a particular flight leg – up-line and down-line swaps required